

PART 1-Factoring (All Methods)

SOLUTIONS

Factor completely

1. $3x^5 - 48x$
2. $3xy - 4 - 6x + 2y$
3. $x^6 - y^6$

PART 2- Zero Product Property

Solve for x using zero product property.

1. $y^2(y - 3)(y^2 - 9) = 0$
2. $t^6 + 9t^2 = 10t^4$

PART 6-Simplify Rational Expressions

1. $\frac{3x-5x^2-2x^3}{6x^2-5x+1}$
2. $\frac{x^4-y^4}{x^4-2x^2y^2+y^4}$

PART 7- Multiply and Divide Rational Expressions

1. $\frac{3x^2-8x+4}{9x^2-4} \times \frac{9x^2-3x-2}{3x^2-5x-2}$
2. $\frac{x^2-4}{2x^2-5x+2} \div \frac{2x^2-3x-2}{4x^2-1}$

PART 8-Adding and Subtracting Rational Expressions

1. $\frac{1}{6a^2} - \frac{1}{2ab} + \frac{3}{8b^2}$
2. $\frac{3}{x^2-5x+6} - \frac{2}{x^2-4}$

PART 9- Complex Fractions

1. $\left(\frac{\frac{z-\frac{1}{z}}{z}}{1-\frac{1}{z}} \right)$
2. $\frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{x}{y} + \frac{y}{x} + 2}$

PART 10-Rational Equations

1. $\frac{3}{x^2-7x+10} + 2 = \frac{x-4}{x-5}$
2. $\frac{u}{u-2} + \frac{30}{u+2} = 9$

PART 17-Radical Equations

1. $\sqrt{x+2} + \sqrt{x-1} = 3$
2. $x - 1 + \sqrt{x^2 + 3} = 0$

Solutions

Part 1 - Factoring

① $3x^5 - 48x$

$$3x(x^4 - 16)$$

$$3x(x^2 + 4)(x^2 - 4)$$

$$3x(x^2 + 4)(x + 2)(x - 2)$$

② $(3xy - 4 - 6x + 2y)$

reorder

$$3xy - 6x + 2y - 4$$

$$3x(y - 2) + 2(y - 2)$$

$$(3x + 2)(y - 2)$$

③ $x^6 - y^6$

$$(x^3 + y^3)(x^3 - y^3)$$

$$(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$$

Part 2 - Zero Product Property

① $y^2(y - 3)(y^2 - 9) = 0$

$$y^2(y - 3)(y + 3)(y - 3) = 0$$

$$y^2 = 0, y - 3 = 0, y + 3 = 0, y - 3 = 0$$

$$y = 0$$

double root

$$y = 3$$

double root

$$y = -3$$

② $t^6 + 9t^2 = 10t^4$

$$-10t^4 \quad -10t^4$$

$$t^6 - 10t^4 + 9t^2 = 0$$

$$t^2(t^4 - 10t^2 + 9) = 0$$

$$t^2(t^2 - 1)(t^2 - 9) = 0$$

$$t^2(t + 1)(t - 1)(t + 3)(t - 3) = 0$$

→ $t^2 = 0, t + 1 = 0, t - 1 = 0, t + 3 = 0, t - 3 = 0$

$$t = 0$$

double root

$$t = -1$$

$$t = 1$$

$$t = -3$$

$$t = 3$$

Part 6 Simplifying Rational Expressions

$$\textcircled{1} \frac{3x - 5x^2 - 2x^3}{6x^2 - 5x + 1} = \frac{-(2x^3 + 5x^2 - 3x)}{6x^2 - 5x + 1}$$

$$= \frac{-x(2x^2 + 5x - 3)}{(3x - 1)(2x - 1)}$$

$$= \frac{-x(\cancel{2x - 1})(x + 3)}{(3x - 1)(\cancel{2x - 1})}$$

$$= \boxed{\frac{-x(x + 3)}{3x - 1}} \quad \text{OR} \quad \boxed{\frac{-x^2 - 3x}{3x - 1}}$$

$$\textcircled{2} \frac{x^4 - y^4}{x^4 - 2x^2y^2 + y^4} = \frac{(x^2 + y^2)(\cancel{x^2 - y^2})}{(x^2 - y^2)(\cancel{x^2 - y^2})}$$

$$= \boxed{\frac{x^2 + y^2}{(x + y)(x - y)}}$$

Part 7 - \times and \div Rational Expressions (3x)

$$\textcircled{1} \frac{3x^2 - 8x + 4}{9x^2 - 4} \times \frac{9x^2 - 3x - 2}{3x^2 - 5x - 2}$$

$$\frac{(3x-2)(x-2)}{(3x+2)(3x-2)} \times \frac{(3x-2)(3x+1)}{(3x+1)(x-2)}$$

$$\boxed{\frac{3x-2}{3x+2}}$$

$$\textcircled{2} \frac{x^2 - 4}{2x^2 - 5x + 2} \div \frac{2x^2 - 3x - 2}{4x^2 - 1}$$

$$\frac{x^2 - 4}{2x^2 - 5x + 2} \cdot \frac{4x^2 - 1}{2x^2 - 3x - 2}$$

$$\frac{(x+2)(x-2)}{(2x+1)(x-2)} \cdot \frac{(2x+1)(2x-1)}{(2x+1)(x-2)} = \boxed{\frac{x+2}{x-2}}$$

Part 8 + and - Rational Expressions

$$\textcircled{1} \quad \frac{1}{6a^2} \cdot \frac{(4b^2)}{(4b^2)} - \frac{1}{2ab} \cdot \frac{(12ab)}{(12ab)} + \frac{3}{8b^2} \cdot \frac{(3a^2)}{(3a^2)} \quad \text{LCD: } 24a^2b^2$$

$$\frac{4b^2}{24a^2b^2} - \frac{12ab}{24a^2b^2} + \frac{9a^2}{24a^2b^2}$$

$$\boxed{\frac{4b^2 - 12ab + 9a^2}{24a^2b^2}}$$

$$\textcircled{2} \quad \frac{3}{x^2 - 5x + 6} - \frac{2}{x^2 - 4} \quad \text{LCD: } (x-3)(x-2)(x+2)$$

$$\frac{(x+2)}{(x+2)} \cdot \frac{3}{(x-3)(x-2)} - \frac{2}{(x+2)(x-2)} \cdot \frac{(x-3)}{(x-3)}$$

$$\frac{3(x+2) - 2(x-3)}{(x+2)(x-2)(x-3)}$$

$$\frac{3x+6-2x+6}{(x+2)(x-2)(x-3)} = \boxed{\frac{x+12}{(x+2)(x-2)(x-3)}}$$

Part 9 Complex Fractions

$$\textcircled{1} \frac{\left(\frac{z}{z} \right) \frac{z}{1} - \frac{1}{z}}{\left(\frac{z}{z} \right) \frac{1}{1} - \frac{1}{z}} = \frac{\frac{z^2-1}{z}}{\frac{z-1}{z}}$$

$$= \frac{z^2-1}{\cancel{z}} \cdot \frac{\cancel{z}}{z-1}$$

$$= \frac{z^2-1}{z-1} = \frac{(z+1)(\cancel{z-1})}{(\cancel{z-1})}$$

$$= \boxed{z+1}$$

$$\textcircled{2} \frac{\left(\frac{y^2}{y^2} \right) \frac{1}{x^2} - \frac{1}{y^2} \left(\frac{x^2}{x^2} \right)}{\frac{\left(\frac{x}{x} \right) \frac{x}{y} + \frac{y}{x} \left(\frac{y}{y} \right) + \frac{2}{1} \left(\frac{xy}{xy} \right)} = \frac{\frac{y^2-x^2}{x^2y^2}}{\frac{x^2+y^2+2xy}{xy}}$$

$$= \frac{y^2-x^2}{x^2y^2} \cdot \frac{\cancel{xy}}{x^2+2xy+y^2}$$

$$= \frac{y^2-x^2}{xy} \cdot \frac{1}{(x+y)(x+y)}$$

$$= \frac{(\cancel{y+x})(y-x)}{(\cancel{xy}) \cdot 1} \cdot \frac{1}{(\cancel{x+y})(x+y)}$$

$$\boxed{\frac{y-x}{xy(x+y)}}$$

Part 10

$$\text{LCD: } (x-5)(x-2)$$

$$\textcircled{1} \quad \frac{3}{x^2-7x+10} + \frac{2}{1} = \frac{x-4}{x-5}$$

$$\left(\frac{3}{(x-5)(x-2)} + 2 \right) = \left(\frac{x-4}{x-5} \right) (x-5)(x-2)$$

$$3 + 2(x-5)(x-2) = (x-4)(x-2)$$

$$3 + 2(x^2 - 7x + 10) = x^2 - 6x + 8$$

$$3 + 2x^2 - 14x + 20 = x^2 - 6x + 8$$

$$\begin{array}{r} 2x^2 - 14x + 23 = \cancel{x^2} - 6x + 8 \\ \underline{-x^2} \quad \quad \quad \underline{-x^2} \end{array}$$

$$\begin{array}{r} x^2 - 14x + 23 = -6x + 8 \\ \quad \quad \quad +6x \quad \quad \quad +6x \end{array}$$

$$\begin{array}{r} x^2 - 8x + 23 = 8 \\ \quad \quad \quad -8 \quad \quad -8 \end{array}$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$x-5=0, x-3=0 \quad \text{ck:}$$

$$\textcircled{x=5}$$

$$\textcircled{x=3}$$

It works.

$$\frac{3}{(5-5)(5-2)} + 2 = \frac{5-4}{5-5}$$

$$\frac{3}{0(-3)} + 2 = \frac{1}{0} \quad \text{undefined}$$

$$\frac{3}{3^2-7(3)+10} + 2 = \frac{3-4}{3-5}$$

$$\frac{3}{9-21+10} + 2 = \frac{-1}{-2}$$

$$\frac{3}{-2} + 2 = \frac{-1}{-2}$$

$$-1\frac{1}{2} + 2 = \frac{1}{2}$$

$$-\frac{1}{2} = -\frac{1}{2}$$

$$\textcircled{2} (u+2)(u-2) \left(\frac{-u}{u-2} + \frac{30}{u+2} \right) = 9(u+2)(u-2) \quad \text{LHD: } (u-2)(u+2)$$

$$u(u+2) + 30(u-2) = 9(u^2-4)$$

$$u^2+2u+30u-60 = 9u^2-36$$

$$\cancel{u^2}+32u-60 = 9u^2-36$$

$$\begin{array}{r} -u^2 \\ \hline 32u-60 = 9u^2-36 \\ -32u \\ \hline -60 = 8u^2-36 \\ +60 \end{array}$$

$$\begin{array}{r} -40 = 8u^2-32u-36 \\ +60 \\ \hline 0 = 8u^2-32u+24 \end{array}$$

$$0 = 8(u^2-4u+3)$$

$$0 = 8(u-3)(u-1)$$

$$u-3=0, \quad u-1=0$$

$$\boxed{u=3}$$

$$\boxed{u=1}$$

then both check!



I'm too lazy!
you can't be!



Do the checks!

Part 17 Radical Equations

Remember, it's
 $(3 - \sqrt{x-1})(3 - \sqrt{x-1})$
expand...

$$\textcircled{1} \quad \sqrt{x+2} + \sqrt{x-1} = 3$$

$$\left(\sqrt{x+2}\right)^2 = \left(3 - \sqrt{x-1}\right)^2$$

$$x+2 = 9 - 6\sqrt{x-1} + (x-1)$$

$$x+2 = 9 - 6\sqrt{x-1} + x-1$$

$$x+2 = 8 - 6\sqrt{x-1} + x$$

$$\cancel{x} - 6 = -6\sqrt{x-1} + \cancel{x}$$

$$\frac{-6}{-6} = \frac{-6\sqrt{x-1}}{-6}$$

$$1 = (\sqrt{x-1})^2$$

$$1 = x-1$$

$$+1 \quad +1$$

$$\boxed{2 = x}$$

Check: $\sqrt{2+2} + \sqrt{2-1} \stackrel{?}{=} 3$

$$\sqrt{4} + \sqrt{1} \stackrel{?}{=} 3$$

$$2 + 1 \stackrel{?}{=} 3$$

$$3 = 3 \checkmark$$

It works! ☺

$$\textcircled{2} \quad \frac{\cancel{x} - 1 + \sqrt{x^2 + 3}}{-x} = 0$$

$$\frac{\cancel{x} - 1 + \sqrt{x^2 + 3}}{+1} = \frac{-x}{+1}$$

$$(\sqrt{x^2 + 3})^2 = (1 - x)^2$$

$$\frac{\cancel{x^2} + 3}{-\cancel{x^2}} = \frac{1 - 2x + \cancel{x^2}}{-\cancel{x^2}}$$

$$\frac{3}{-1} = \frac{1 - 2x}{-1}$$

$$\frac{2}{-2} = \frac{\cancel{-2x}}{-\cancel{2}}$$

$$\boxed{-1 = x}$$

Check:

$$-1 - 1 + \sqrt{(-1)^2 + 3} \stackrel{?}{=} 0$$

$$-2 + \sqrt{1 + 3} \stackrel{?}{=} 0$$

$$-2 + \sqrt{4} \stackrel{?}{=} 0$$

$$-2 + 2 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

It works! 😊